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Scattering from elastic sea beds: First-order theory

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Abstract

A perturbation model for high-frequency sound scattering from an irregular elastic sea bed is considered. The sea bed is assumed homogeneous on the average and two kinds of irregularities are assumed to cause scattering: roughness of the water-sea bed interface and volume inhomogeneities of the sediment mass density and the speeds of compressional and shear waves. The first-order small perturbation approximation is used to obtain expressions for the scattering amplitude and bistatic scattering strength. The angular dependence of the scattering strength is calculated for sedimentary rock and the influence of shear elasticity is examined by comparison with the case of a fluid bottom. Shear effects are shown to be strong and complicated.

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INTRODUCTION

The Born approximation (first-order perturbation theory) has been applied to sound scattering by rough, elastic sea beds by Lapin [23, 24], Dacol and

Berman [4], and Kuo [19]. Kuperman and Schmidt [21] have used the Born approximation to obtain the coherent field in layered elastic sea beds with rough interface. Essen [5] has also used the Born approximation to treat rough sea bed types ranging from very soft sediment to basalt. This previous work on interface scattering leaves open the question of the effect of shear on scattering from within the volume of the sea bed. Numerous authors have applied the Born approximation to volume scattering in fluid sediments [7, 8, 9, 11, 6, 18, 25, 26, 27, 28, 32], and these results show that volume scattering can be an important mechanism. Crowther [3], Ivakin [10, 12, 14, 15, 16] and Kuo [20] have applied the Born approximation to the joint roughness-inhomogeneity problem for fluid sediments. While Ivakin [13] has given a formal Born approximation treatment of the volume scattering problem for elastic sea beds, there has been no joint treatment of roughness and volume scattering for elastic sea beds. The purpose of the present article is to put the rather complicated roughness and volume scattering theoretical results into a common and general format that is straightforward from a numerical point of view and that lends itself to systematic study of these two processes. A subsequent article [17] uses this new formulation in an evaluation of the importance of shear effects in scattering from the sea bed.

1 FORMULATION

Much previous work on acoustic scattering from the sea bed assumes that the sediment can be treated as a lossy fluid, in which case scattering at the interface is controlled by the discontinuities in density and compressional wave speed. Volume scattering is controlled by inhomogeneities in the same two parameters. We will refer to these controlling factors as *scattering mechanisms.* Note that the choice of compressional wave speed as a bulk parameter is somewhat arbitrary; the bulk modulus or some other independent parameter could be used. When an elastic model is used for the sediment, additional degrees of freedom appear which introduce additional scattering mechanisms. In particular, one must now consider the discontinuities and inhomogeneities in shear wave speed as well as the density and compressional wave speed. Furthermore, the scattering mechanisms may generate shear waves within the sediment. As illustrated by Fig. 1, this leads to a number of scattering mechanisms. For volume scattering, for example, one mechanism is scatter-

ing of a compressional wave by a density fluctuation, leading to conversion to a shear wave. This shear wave can be converted to a water-borne compressional wave upon encountering the interface. If the shear wave is polarized such that the particle motion is in a vertical plane, conversion is possible even for the flat interface. As a result, this scattering mechanism is evident in a first-order perturbation treatment. Scattering to the other possible polarization (with particle motion in a horizontal plane) will only be evident at higher order, as such a wave suffers total internal reflection at a horizontal, flat interface.

1.1 Basic Equations

Figure 2 depicts the general physical situation to be considered. A rough interface described by the random function

$$x_3 = \zeta(\mathbf{R}) \tag{1}$$

separates a lossless, homogeneous fluid from an inhomogeneous elastic medium. In Eq. 1 and following equations, upper-case letters will be used to denote two-dimensional vectors and lower-case letters will denote three-dimensional vectors. Thus, the three-dimensional position vector, $\mathbf{r}$, is decomposed into its transverse and vertical components as

$$\mathbf{r} = (\mathbf{R}, x_3) \quad , \tag{2}$$

where

$$\mathbf{R} = (x_1, x_2) \quad . \tag{3}$$

It is convenient to describe wave motion in terms of the particle displacement field, $\mathbf{u}(\mathbf{r})$. This field, in turn, is expressed in terms of scalar and vector potentials [2]

$$\mathbf{u}(\mathbf{r}) = \nabla\phi(\mathbf{r}) + \nabla \times \vec{\psi}(\mathbf{r}) \quad . \tag{4}$$

These potentials describe two different types of waves, compressional and shear, supported by an elastic medium. As the water does not support shear, the vector potential, $\vec{\psi}(\mathbf{r})$, in the water is taken to be zero.

The general equation obeyed by the displacement field is

$$\omega^2\rho u_i+\partial_i(\lambda\nabla\cdot\mathbf{u})+\sum_j\partial_j\left(\mu(\partial_i u_j+\partial_j u_i)\right)=0\ . \tag{5}$$

where the density, ρ, and the Lamé parameters, λ and μ, can be functions of position, and $\partial_i=\frac{\partial}{\partial x_i}$. In the water, μ is taken to be zero. The time dependence of all field quantities is assumed to be $\exp(-i\omega t)$.

The boundary conditions on the interface ($x_3=\zeta(\mathbf{R})$) are continuity of the normal component of displacement

$$\mathbf{u}^{(f)}\cdot\mathbf{N}=\mathbf{u}\cdot\mathbf{N}\ \ , \tag{6}$$

and continuity of normal tractions for the stress tensor, σ_{ij},

$$\sigma_{ij}=\lambda\delta_{ij}\nabla\cdot\mathbf{u}+\mu(\partial_i u_j+\partial_j u_i)\ \ , \tag{7}$$

that is,

$$\sum_j\sigma_{ij}^{(f)}N_j=\sum_j\sigma_{ij}N_j\ \ , \tag{8}$$

where the superscript , f, denotes the fluid medium (water) and no superscript is used for the elastic medium (sea bed material). The vector, $\mathbf{N}$, is the (unnormalized) interface normal,

$$\mathbf{N}=-\nabla\zeta(\mathbf{R})+\mathbf{e_3}\ \ , \tag{9}$$

where $\mathbf{e_3}$ is the unit vector in the vertical direction.

As the traction boundary condition, Eq. 8, is actually three equation, there are a total of four equations that must be satisfied. There are also four unknowns: the scattered compressional field in the water and the three scattered fields in the sea bed material (one compressional field and two shear fields having horizontal and vertical polarizations).

1.2 Sea Bed Parameters

As mentioned above, other parameters can be used instead of λ and μ. In the present case, compressional and shear wave speeds, c_p and c_t, will be employed rather than the Lamé parameters. The various parameters are related as follows:

$$\lambda = \rho(c_p^2 - 2c_t^2), \quad \mu = \rho c_t^2. \tag{10}$$

Subscripts, p and t, here and below denote compressional (longitudinal) and transverse (shear) waves, respectively. In the water, c_t and μ are taken to be zero.

The parameters will be divided into two classes: those representing the average, non-fluctuating values of bulk properties such as density and wave speeds, and those describing the random fluctuation of bulk properties and interface relief. In the context of perturbation theory, the zeroth-order solution is that solution corresponding to the average properties with a planar interface. The first-order correction to this solution, for the scattered field, is linear in the fluctuating parameters, including the relief of the interface.

It is convenient to use dimensionless ratios of the average parameters. For simplicity, average wave speeds and density are simply denoted c_p, c_t, and ρ, although these symbols were earlier used for the position-dependent quantities, that is, the average value plus fluctuations. This simplified notation will be employed in the remainder of this article. Thus, define

$$a_f = 1 \quad , a_p = c_p/c_f \quad , a_t = c_t/c_f \quad , a_\rho = \rho/\rho_f \quad . \tag{11}$$

The trivial ratio a_f is defined for convenience in later expressions. The compressional wave speed ratio, a_p, is the ratio of the average sediment compressional wave speed and the water compressional wave speed. This ratio is complex, with imaginary part determining absorption loss. The shear speed ratio, a_t, is likewise the ratio of the average complex sediment shear speed, and the compressional wave speed in the water and the density ratio, a_ρ, is the ratio of the average sediment mass density to the density of the overlying water.

Interface roughness will be described by the spectral density function, $\Phi^{(r)}(\mathbf{K})$, where $\mathbf{K}$ is the two-dimensional spectral argument. In the Born approximation, this is a sufficient description of the statistics of the random interface. In other approximations, such as higher-order perturbation theory, additional statistical information must be provided by specifying higher-order statistics or by assuming Gaussian statistics.

A description of the statistics of volume inhomogeneities is given by a matrix of spatial (3-dimensional) cross-spectra. These are Fourier transforms of cross-correlation functions for the relative fluctuations $\epsilon_\rho = \Delta\rho/\rho, \epsilon_p =$

$\Delta c_p/c_p, \epsilon_t = \Delta c_t/c_t$. These relative fluctuations will be collectively designated ϵ_β with $\beta = \rho, p, t$. In this notation, the cross-spectra are $\Phi^{(v)}_{\beta\beta'}(\mathbf{K},q)$ where $\mathbf{K}$ and q are the horizontal and vertical components of the three-dimensional spectral argument. Again, these spectra provide a sufficient description of the inhomogeneities in the Born approximation.

2 THEORETICAL RESULTS

The end result of the formulas to be presented is the scattering coefficient, σ, or scattering cross section per unit area per unit solid angle. The term "cross section" will be used for brevity, even though this quantity is dimensionless. The quantity $10 \log \sigma$ is commonly called the scattering strength and is expressed in decibels. The scattering cross section characterizes the frequency-angular distribution of the mean intensity of field fluctuations in the Fraunhofer zone (or far zone, relative to a scattering surface or volume). In the far zone, the scattered acoustic wave can be approximated as a plane wave. Likewise, it is implicit in the definition of scattering strength that the scattering interface is in the far zone of the source, so that the incident field can also be approximated by a plane wave. The scattering cross section is assumed here to be the sum of contributions from interface roughness and volume inhomogeneity. In the first-order approximation used here, this will be the case if the interface relief is uncorrelated with the volume inhomogeneities [3, 10, 14, 20].

$$\sigma(\mathbf{K}_s, \mathbf{K}_i) = \sigma_r(\mathbf{K}_s, \mathbf{K}_i) + \sigma_v(\mathbf{K}_s, \mathbf{K}_i) \tag{12}$$

Expressions for these two components are given without derivation in the two following subsections. The appendices give outlines of the derivation of $\sigma_r(\mathbf{K}_s, \mathbf{K}_i)$ and $\sigma_v(\mathbf{K}_s, \mathbf{K}_i)$ using unified notation. In addition, the volume scattering formalism of Ref. [13] is generalized to include a wider class of scattering mechanisms. The formulas presented in this section are written in a form convenient for coding in higher-level matrix-oriented computational language.

2.1 Relevant Plane Waves

It is convenient to present a solution of the first-order scattering problem in terms of solutions of the unperturbed wave equations, that is, the plane waves propagating in the up and down directions,

$$\exp(i\mathbf{k}_\alpha^\pm \cdot \mathbf{r}) \, , \alpha = f, \; p, \; t \;\; ,$$

with the subscripts α denoting the wave vectors for compressional waves in the water, compressional waves in the sea bed, and shear waves in the sea bed, respectively. The superscripts $+$ and $-$ denote the direction of the wave propagation, up and down, respectively.

It is also appropriate to define the general three-dimensional wave vectors through the transverse components of the incoming and outgoing wave vectors

$$\mathbf{k}_\alpha^\pm(\mathbf{K}) = (\mathbf{K}, \pm k_f \nu_\alpha(\mathbf{K})) \; , \alpha = f, \; p, \; t \tag{13}$$

The unperturbed wave equations obeyed by the potentials determine the relation between the transverse and vertical components of the wavevectors:

$$\nu_\alpha(\mathbf{K}) = \sqrt{a_\alpha^{-2} - K^2/k_f^2} \; , \alpha = f, p, t \;\; . \tag{14}$$

The speed ratios, a_α, were defined earlier.

The following unit vectors specifying the directions of propagation are also useful.

$$\mathbf{e}_\alpha^\pm(\mathbf{K}) = \mathbf{k}_\alpha^\pm(\mathbf{K})/k_\alpha \;\; , \tag{15}$$

where

$$k_\alpha = \frac{\omega}{c_\alpha} \;\; , \alpha = f, p, t, \tag{16}$$

are the wave numbers, respectively, of compressional waves in water, compressional waves in the sea bed, and shear waves in the sea bed.

There are two plane-wave shear polarizations to consider, both having particle displacement normal to the direction of propagation. The shear wave having particle displacement in the direction

$$\mathbf{e}_h^\pm(\mathbf{K}) = \mathbf{e}_3 \times \mathbf{K}/K = (-K_2/K, K_1/K, 0), \tag{17}$$

will be referred to as "horizontally polarized" as the particle displacement is in a horizontal plane. In this case, the polarization vector does not depend

upon whether propagation is upward or downward, but the superscripts $\pm$ are used for consistency with the vertical polarization case. A plane shear wave having transverse wave vector, $\mathbf{K}$, and particle displacement in the direction of the unit vector

$$\mathbf{e}_v^{\pm}(\mathbf{K}) = \mathbf{k}_t^{\pm}(\mathbf{K}) \times \mathbf{e}_h^{\pm}(\mathbf{K})/k_t = \mp \mathbf{K} a_t \nu_t(\mathbf{K})/K + \mathbf{e}_3 K/k_t \ , \tag{18}$$

will be referred to as "vertically polarized", as the displacement is in a vertical plane. In this case, the polarization vector depends upon whether the wave is propagating upward or downward. Scattering by interface roughness and inhomogeneities within the seabed causes conversion between the wave types defined above. For example, while incidence of a plane compressional wave in the water on a flat interface only gives rise to a vertically polarized shear wave, scattering by interface roughness causes conversion of compressional energy in the water to both the vertical and horizontal shear polarizations. Scattering by inhomogeneities within the seabed causes conversion between all three wave types.

The above expressions for wave vectors and polarization vectors are written in terms of a general transverse wave vector, $\mathbf{K}$, for later use in the theoretical development. In computing the scattering cross section, however, one must take $\mathbf{K} = \mathbf{K}_{i,s}$. In this case, it is convenient to define the transverse components in terms of the angular coordinates of interest:

$$\mathbf{K}(\theta, \phi) = (k_f \cos\theta \cos\phi, k_f \cos\theta \sin\phi) \ , \tag{19}$$

and

$$\mathbf{K}_{i,s} = \mathbf{K}(\theta_{i,s}, \phi_{i,s}) \ , \tag{20}$$

where θ_i and θ_s, are the grazing angles of the incident and scattered acoustic waves, respectively, and ϕ_s and ϕ_i are the azimuthal angles of the incident and scattered waves, respectively. The unit vectors can then be expressed as follows:

$$\mathbf{e}_\alpha^{\pm}(\mathbf{K}) = a_\alpha[\cos\theta\cos\phi, \cos\theta\sin\phi, \pm\nu_\alpha(\mathbf{K})] \ , \tag{21}$$

$$\mathbf{e}_h^{\pm}(\mathbf{K}) = [-\sin\phi, \cos\phi, 0] \ , \tag{22}$$

$$\mathbf{e}_v^{\pm}(\mathbf{K}) = a_t[\mp\nu_t(\mathbf{K})\cos\phi, \mp\nu_t(\mathbf{K})\sin\phi, \cos\theta] \ , \tag{23}$$

with

$$\nu_\alpha(\mathbf{K}) = \sqrt{a_\alpha^{-2} - \cos^2\theta}. \tag{24}$$

In Eqs. 21-24, subscripts i and s must be added to $\mathbf{K}$ and to the angles θ and ϕ as appropriate.

2.2 Scattering due to Interface Roughness

The derivation of the results presented here is given in Appendix B. The scattering cross section for interface roughness is of the form

$$\sigma_r(\mathbf{K}_s, \mathbf{K}_i) = |H_f(\mathbf{K}_s, \mathbf{K}_i)|^2 \Phi^{(r)}(\mathbf{K}_s - \mathbf{K}_i) \ , \tag{25}$$

where $\Phi^{(r)}$ is the roughness spectrum defined in Eq. 50, and $H_f(\mathbf{K}_s, \mathbf{K}_i)$ is the first element of the column matrix

$$H(\mathbf{K}_s, \mathbf{K}_i) = \begin{pmatrix} H_f(\mathbf{K}_s, \mathbf{K}_i) \\ H_p(\mathbf{K}_s, \mathbf{K}_i) \\ H_v(\mathbf{K}_s, \mathbf{K}_i) \\ H_h(\mathbf{K}_s, \mathbf{K}_i) \end{pmatrix} . \tag{26}$$

As explained in Appendix B, the elements of $H_f(\mathbf{K}_s, \mathbf{K}_i)$ determine the plane-wave spectra of energy scattered into the water, into compressional waves in the sea bed, and into vertically and horizontally polarized shear waves in the sea bed. This matrix, in turn, is computed as follows:

$$H(\mathbf{K}_s, \mathbf{K}_i) = k_f^2 Y_1(\mathbf{K}_s)[P^{(3)}(\mathbf{K}_s)]^{-1} B(\mathbf{K}_s, \mathbf{K}_i) D_0(\mathbf{K}_i) \ . \tag{27}$$

In this equation, $D_0(\mathbf{K}_i)$ is a five-row column vector comprised of the four transformation (reflection-transmission) coefficients contained in the column vector, $S_0(\mathbf{K}_i)$, and supplemented with unity in the last row.

$$D_0(\mathbf{K}_i) = \begin{pmatrix} S_0(\mathbf{K}_i) \\ --- \\ 1 \end{pmatrix} , \tag{28}$$

An expression for computation of $S_0(\mathbf{K}_i)$ will be given later. Matrix $B(\mathbf{K}_s, \mathbf{K}_i)$ is

$$B(\mathbf{K}_s, \mathbf{K}_i) = k_f^{-1}(K_{s1} - K_{i1}) E^{(1)}(\mathbf{K}_i) + k_f^{-1}(K_{s2} - K_{i2}) E^{(2)}(\mathbf{K}_i) - Y_2(\mathbf{K}_i) E^{(3)}(\mathbf{K}_i) \ . \tag{29}$$

Note that the factors $k_f^{-1}(K_{sn} - K_{in})$, $n = 1, 2$, are dimensionless and can be expressed conveniently in terms of the angles defined in Eq. 19. The matrices Y_1 and Y_2 are

$$Y_1(\mathbf{K}) = \begin{pmatrix} \nu_f(\mathbf{K}) & 0 & 0 & 0 \\ 0 & \nu_p(\mathbf{K}) & 0 & 0 \\ 0 & 0 & \nu_t(\mathbf{K}) & 0 \\ 0 & 0 & 0 & \nu_t(\mathbf{K}) \end{pmatrix} , \tag{30}$$

$$Y_2(\mathbf{K}) = \begin{pmatrix} \nu_f(\mathbf{K}) & 0 & 0 & 0 & 0 \\ 0 & -\nu_p(\mathbf{K}) & 0 & 0 & 0 \\ 0 & 0 & -\nu_t(\mathbf{K}) & 0 & 0 \\ 0 & 0 & 0 & -\nu_t(\mathbf{K}) & 0 \\ 0 & 0 & 0 & 0 & -\nu_f(\mathbf{K}) \end{pmatrix} . \tag{31}$$

There are three matrices $E^{(n)}(\mathbf{K})$, defined for each of the coordinate indices, $n = 1, 2, 3$

$$E^{(n)}(\mathbf{K}) = \left(\begin{array}{cccc|c} P_{f1}^{(n)}(\mathbf{K}) & P_{p1}^{(n)}(\mathbf{K}) & P_{v1}^{(n)}(\mathbf{K}) & P_{h1}^{(n)}(\mathbf{K}) & Q_1^{(n)}(\mathbf{K}) \\ P_{f2}^{(n)}(\mathbf{K}) & P_{p2}^{(n)}(\mathbf{K}) & P_{v2}^{(n)}(\mathbf{K}) & P_{h2}^{(n)}(\mathbf{K}) & Q_2^{(n)}(\mathbf{K}) \\ P_{f3}^{(n)}(\mathbf{K}) & P_{p3}^{(n)}(\mathbf{K}) & P_{v3}^{(n)}(\mathbf{K}) & P_{h3}^{(n)}(\mathbf{K}) & Q_3^{(n)}(\mathbf{K}) \\ P_{f4}^{(n)}(\mathbf{K}) & P_{p4}^{(n)}(\mathbf{K}) & P_{v4}^{(n)}(\mathbf{K}) & P_{h4}^{(n)}(\mathbf{K}) & Q_4^{(n)}(\mathbf{K}) \end{array}\right) , \tag{32}$$

where the dashed vertical line in Eq.32 separates it into the the 4×4 matrices, $P^{(n)}(\mathbf{K})$, and the column matrices, $Q^{(n)}(\mathbf{K})$. Note that $P^{(3)}(\mathbf{K}_s)$ appears in Eq. 27. The elements of $P^{(n)}(\mathbf{K})$ and $Q^{(n)}(\mathbf{K})$ are

$$P_{fm}^{(n)}(\mathbf{K}) = \delta_{mn} , \quad m = 1, 2, 3 , \tag{33}$$

$$P_{f4}^{(n)}(\mathbf{K}) = e_{fn}^{+}(\mathbf{K}) , \tag{34}$$

$$P_{pm}^{(n)}(\mathbf{K}) = a_\rho[-\delta_{mn} + 2(a_t/a_p)^2(\delta_{mn} - e_{pm}^{-}e_{pn}^{-})] , \quad m = 1, 2, 3 , \tag{35}$$

$$P_{p4}^{(n)}(\mathbf{K}) = -a_p^{-1}e_{pn}^{-}(\mathbf{K}) , \tag{36}$$

$$P_{(v,h)m}^{(n)}(\mathbf{K}) = -a_\rho[e_{tm}^{-}e_{(v,h)n}^{-} + e_{tn}^{-}e_{(v,h)m}^{-}] , \quad m = 1, 2, 3 , \tag{37}$$

$$P_{(v,h)4}^{(n)}(\mathbf{K}) = -a_t^{-1}e_{(v,h)n}^{-}(\mathbf{K}) , \tag{38}$$

$$Q_m^{(n)}(\mathbf{K}) = \delta_{mn} \ , \quad m = 1, 2, 3 \ , \tag{39}$$

$$Q_4^{(n)}(\mathbf{K}) = e_{fn}^{-}(\mathbf{K}) \ , \tag{40}$$

where δ_{nm} is the Kronecker delta function and the unit vectors defined earlier are employed.

The transformation matrix, $S_0(\mathbf{K}_i)$, is computed as follows:

$$S_0(\mathbf{K}_i) = -[P^{(3)}(\mathbf{K}_i)]^{-1} Q^{(3)}(\mathbf{K}_i) \ . \tag{41}$$

The four elements of this matrix are denoted

$$S_0(\mathbf{K}_i) = \begin{pmatrix} W_f(\mathbf{K}_i) \\ W_p(\mathbf{K}_i) \\ W_v(\mathbf{K}_i) \\ W_h(\mathbf{K}_i) \end{pmatrix} , \tag{42}$$

and are the in-water reflection coefficient for compressional waves, and the transmission coefficients for compressional waves and the two shear waves in the sea bed (see Appendix B). Alternative expressions for these coefficients are given in Ref. [2]. Note that $W_h(\mathbf{K}_i) = 0$. These coefficients are also required for the volume scattering computation.

2.3 Scattering due to Volume Inhomogeneity

The scattering cross section for volume inhomogeneity is of the form

$$\sigma_v(\mathbf{K}_s, \mathbf{K}_i) = -\frac{\pi k_f^4 a_\rho^2}{2} Im \left(\sum_{\eta,\beta,\eta',\beta'} d_{\eta\beta} d^*_{\eta'\beta'} \frac{\Phi_{\beta\beta'}\left((\mathbf{q}_\eta + \mathbf{q}^*_{\eta'})/2\right)}{(q_{\eta 3} - q^*_{\eta' 3})} \right) , \tag{43}$$

where Im designates the imaginary part of the complex quantity and $\Phi_{\beta\beta'}$ is the is the matrix of cross spectra for volume inhomogeneities defined in Eq. 51. Thus, the β and β' sums run over the three types of inhomogeneity: density, compressional wave speed, and shear wave speed ($\beta, \beta' = \rho, p, t$). Generally, these fluctuations are expected to be strongly correlated, thus the $\beta \neq \beta'$ cross-terms are important. The η and η' sums run over the four types of wave conversion caused by volume scattering: 1. compressional to compressional ($\eta = pp$), 2. shear to compressional ($\eta = pt$), 3. compressional to shear ($\eta = tp$), and 4. shear to shear ($\eta = tt$). Thus, we can put

$$\eta = \alpha\alpha',$$

where α and α' run over the two types of waves ($\alpha, \alpha' = p, t$). As noted earlier, volume inhomogeneities cause conversion to both shear polarizations, but upgoing shear waves with horizontal particle motion suffer total reflection at the interface and do not contribute to the scattering cross section in first order. The scattering vectors for the four relevant types of conversion corresponding to different channels of scattering are

$$\mathbf{q}_{\alpha\alpha'} = \mathbf{k}_{\alpha}^{+}(\mathbf{K}_s) - \mathbf{k}_{\alpha'}^{-}(\mathbf{K}_i) \tag{44}$$

These vectors give the change in wave vector for the corresponding conversion and appear as arguments in the cross spectra in Eq. 43. All of these vectors have the same transverse components

$$\mathbf{q}_{\eta} = (\mathbf{K}_s - \mathbf{K}_i, q_{\eta 3}) \ . \tag{45}$$

The vertical components are

$$q_{\alpha\alpha' 3} = k_f[\nu_{\alpha}(\mathbf{K}_s) + \nu_{\alpha'}(\mathbf{K}_i)] \ , \tag{46}$$

The coefficients $d_{\eta\beta}$ are

$$d_{\eta\beta} = w_{\eta} D_{\eta\beta} \tag{47}$$

where

$$w_{\alpha\alpha'} = W_{\alpha}(\mathbf{K}_s) W_{\alpha'}(\mathbf{K}_i), \ \ \alpha, \alpha' = p, t \tag{48}$$

with $W_t = W_v$. The coefficients $D_{\alpha\beta}$ are elements of the 3-column matrix

$$D = (D_{\rho} | D_p | D_t)$$

where

$$D_{\rho} = \begin{pmatrix} a_p^{-2} - 2a_t^2 a_p^{-4} + 2a_t^2 b_{pp}^2 - b_{pp} \\ -b_{pv} + 2a_t^2 b_{pv} b_{pt} \\ b_{vp} - 2a_t^2 b_{vp} b_{pt} \\ b_{vv} - a_t^2 b_{vv} b_{tt} + a_t^2 b_{vt} b_{tv} \end{pmatrix}$$

Eq No?

$$D_p = (2, 0, 0, 0)^T$$

$$D_t = -2a_t^2 \begin{pmatrix} 2a_p^{-2} - 2b_{pp}^2 \\ -2b_{pt}b_{pv} \\ 2b_{tp}b_{vp} \\ b_{vv}b_{tt} - b_{vt}b_{tv} \end{pmatrix}$$

Eq No. ?

Here

$$b_{\alpha\alpha'} = \mathbf{k}_\alpha^+(\mathbf{K}_s) \cdot \mathbf{k}_{\alpha'}^-(\mathbf{K}_i)/k_f^2 \tag{49}$$

where $\alpha, \alpha' = p, t, v$ and the wave vectors $\mathbf{k}_\alpha^\pm(\mathbf{K})$ are defined in Eq. 13.

3 CALCULATIONS AND DISCUSSION

3.1 Model Parameters

To illustrate some of the effects of shear elasticity on sea bed scattering, calculations will be presented for a single set of parameters, with average physical parameters appropriate to sedimentary rock [5]. These are density ratio, $a_\rho = 2.5$, compressional speed ratio, $a_p = 2.3 - 0.004i$, and shear speed ratio, $a_t = 1.3 - 0.22i$. The water sound speed is taken to be 1500 m/s and the frequency is taken to be 1 kHz. As noted later, the roughness and volume fluctuation parameters used are such as to yield essentially no frequency dependence.

The roughness spectrum is taken to be of the form

$$\Phi^{(r)}(\mathbf{K}) = \frac{(2/\pi)A^{(r)}}{(K_0^2 + K^2)^2} \tag{50}$$

Equation 50 assumes that roughness statistics are stationary and isotropic, with a spectrum that obeys a power law for $K_0^2 \ll K^2$. The parameter K_0 is an inverse correlation scale for roughness in the transverse directions. The roughness spectral parameters were assigned the following values: $K_0 = 10^{-2}\mathrm{m}^{-1}$ and $A^{(r)} = 10^{-5}$. The form of the spectrum and parameter K_0 were chosen so that the scattering cross section is essentially independent of frequency except for scattering very close to the specular direction. The parameter $A^{(r)}$ is dimensionless and was chosen arbitrarily to yield scattering strengths that are, except near the specular direction, within the likely region of validity of the Born approximation. The factor $2/\pi$ is included in Eq.50 for convenience [17].

The volume inhomogeneity spectra were taken to be of the form

$$\Phi^{(v)}_{\beta\beta'}(\mathbf{K},q) = \frac{\delta_{\beta\beta'} A^{(v)}/(2\pi)}{(K^2 + q^2 + q_0^2)^{3/2}}. \tag{51}$$

with $q_0 = 10^{-2}\mathrm{m}^{-1}$, $A^{(v)} = 10^{-5}$. These spectra correspond to isotropic uncorrelated inhomogeneities. The form of the spectra and parameter q_0 are chosen so that the scattering cross section is essentially independent of frequency for all scattering directions. The parameter $A^{(v)}$ is dimensionless and was chosen under the same rationale as $A^{(r)}$ with a factor $1/(2\pi)$ included for convenience [17].

Because of the Kroneker delta used in Eq. 51, all the nondiagonal elements of the correlation matrix vanish. In this "uncorrelated" case, the volume component of the scattering cross section, according to Eq.43, will be simply the sum of three components

$$\sigma_v = \sum_{\beta} \sigma_\beta \tag{52}$$

where σ_β correspond to contributions of fluctuations of the three different bulk parameters ($\beta = \rho, c_p, c_t$). Calculations are presented below for each of the terms in Eq. 52. For example, σ_ρ is considered for the case in which there are density fluctuations but no compressional or shear speed fluctuations.

3.2 Numerical Results

The parameters specified above were used in computing angular dependencies of the backscattering strength. The results of these calculations are shown in Fig. 3 which shows the relative contributions of different types of perturbation for the elastic case and compares them to the fluid case. As three different types of volume fluctuation are considered, this comparison involves a set of seven different curves that includes three pairs comparing the elastic and fluid cases for scattering by roughness, by density fluctuations, and by compressional wave speed fluctuations. The seventh curve is for scattering by shear wave speed fluctuations, which have no fluid counterpart.

The results show that for sedimentary rock the effects of shear elasticity on both roughness and volume scattering are very strong. As the average parameters are taken from Essen [5], the roughness scattering results are

equivalent to his. Figure 3 shows that roughness backscattering strength exhibits marked dips at grazing angles immediately below the critical angles for shear and compressional waves (39.7° and 64.2°, respectively). The inclusion of lossy elasticity is seen to decrease the roughness scattering strength for certain angles compared to the fluid case in which the shear speed is set to zero. But, at the same time, it somewhat enhances scattering for other angles (in the 40° - 60° range). Clearly, treating sedimentary rock as a fluid, albeit a massive and stiff fluid, is a poor approximation.

Figure 3 shows that, in contrast to the case of roughness scattering, elasticity greatly enhances volume backscattering in rock for grazing angles smaller than the shear wave critical angle for all the types of volume fluctuations, but can cause a great reduction in scattering from compressional speed fluctuations for grazing angles in the 40° - 60° range. Elasticity effects for volume scattering are much larger than the effects for roughness scattering.

As these results show, the effects of elasticity on roughness and volume backscattering for rock are strong and complicated. A more detailed numerical study of the effects of elasticity for different sea bed types, different sets of statistical parameters, and different types of geometry (both monostatic and bistatic cases) will be given in a subsequent article [17].

4 CONCLUSIONS

First-order perturbation theory has been used to develop expressions for the scattering strength of elastic sea beds. Both roughness and volume components of scattering are included. The roughness component of the scattering strength is related to the spatial two-dimensional spectrum of roughness. The volume component is expressed through three-dimensional cross-spectra for spatial fluctuations of density, compressional and shear wave speed. Shear effects on both roughness and volume scattering are complicated and strong, at least for the case of backscattering from the sedimentary rock taken as an example for calculations. Generally, an assessment of shear effects for various sea bed types requires more detailed numerical study, using available data on sea bed parameters including their statistics.

Acknowledgments

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Appendix A: Scattering Amplitudes

Regardless of the complexity of the random sea bed, whether scattering is due to roughness or volume inhomogeneity, the scattered field in the water above the highest point on the interface can be expressed as a superposition of plane waves. In the Born approximation, plane-wave superpositions are also used to describe the fields in the sediment. If the incident field is a plane wave of unit amplitude having wave vector with transverse component, $\mathbf{K}_i$, then the scattered potential in the water is

$$\phi_f(\mathbf{r}) = \int \frac{d^2K}{k_f \nu_f(\mathbf{K})} A_f(\mathbf{K}, \mathbf{K}_i) \exp(i\mathbf{k}_f^+(\mathbf{K}) \cdot \mathbf{r}) \quad , \tag{53}$$

where $A_f(\mathbf{K}, \mathbf{K}_i)$ is the scattering amplitude [13], related to the transition matrix ("T matrix") [33, 29] as follows:

$$A_f(\mathbf{K}, \mathbf{K}_i) = k_f \nu_f(\mathbf{K}) T(\mathbf{K}, \mathbf{K}_i) \quad . \tag{54}$$

The potential, $\phi_f(\mathbf{r})$, does not include the incident field but does include all other portions of the field (coherent and incoherent). The scattering amplitude obeys the reciprocity relation

$$A_f(\mathbf{K}, \mathbf{K}_i) = A_f(-\mathbf{K}_i, -\mathbf{K}) \quad . \tag{55}$$

Similarly, the total scalar potential in the sea bed is expressed as

$$\phi_p(\mathbf{r}) = \int \frac{d^2K}{k_f \nu_p(\mathbf{K})} A_p(\mathbf{K}, \mathbf{K}_i) \exp(i\mathbf{k}_p^-(\mathbf{K}) \cdot \mathbf{r}) \quad , \tag{56}$$

and the vector potential in the sediment is

$$\vec{\psi}(\mathbf{r}) = \int \frac{d^2K}{k_f \nu_t(\mathbf{K})} \mathbf{A}_t(\mathbf{K}, \mathbf{K}_i) \exp(i\mathbf{k}_t^-(\mathbf{K}) \cdot \mathbf{r}) \quad . \tag{57}$$

where

$$\mathbf{A}_t(\mathbf{K}, \mathbf{K}_i) = \mathbf{e}_h^-(\mathbf{K}) A_v(\mathbf{K}, \mathbf{K}_\mathrm{i}) - \mathbf{e}_v^-(\mathbf{K}) A_h(\mathbf{K}, \mathbf{K}_\mathrm{i}) \tag{58}$$

When the in-water scattering amplitude has been found, all the basic characteristics of the scattered field can be determined. For example, the coherent reflection coefficient is defined by the following equation for the averaged scattering amplitude, or its coherent component

$$< A_f(\mathbf{K}, \mathbf{K}_i) >= \delta(\mathbf{K} - \mathbf{K}_i)\overline{W}_f(\mathbf{K}_i)\nu_f(\mathbf{K}_i). \qquad (59)$$

Generally, the coherent reflection coefficient, $\overline{W}_f(\mathbf{K}_i)$, is not equal to the zeroth-order reflection coefficient, but this distinction is not significant in the present problem.

The scattering amplitude will be used to obtain the scattering cross section through the equation [33, 31, 29, 1]

$$< A'_f(\mathbf{K}_s, \mathbf{K}_i)A'^*_f(\mathbf{K}'_s, \mathbf{K}_i) >= \delta(\mathbf{K}_s - \mathbf{K}'_s)\sigma(\mathbf{K}_s, \mathbf{K}_i) \ , \qquad (60)$$

where $A'_f(\mathbf{K}_s, \mathbf{K}_i) = A_f(\mathbf{K}_s, \mathbf{K}_i) - < A_f(\mathbf{K}_s, \mathbf{K}_i) >$ is the incoherent component of the scattering amplitude and it is assumed that the statistics of the random sea bed are stationary in the the two transverse (horizontal) coordinates. The water column is assumed to be homogeneous, supporting plane waves, and the scattering cross section is assumed to be the sum of contributions from interface roughness and volume inhomogeneity.

Appendix B: Interface Scattering

The incoherent roughness scattering amplitudes in the Born approximation have the general form

$$A'_\alpha(\mathbf{K}_s, \mathbf{K}_i) = iH_\alpha(\mathbf{K}_s, \mathbf{K}_i)Z(\mathbf{K}_s - \mathbf{K}_i) \ , \alpha = f, p, v, h \quad , \qquad (61)$$

where $Z(\mathbf{K})$ is the Fourier transform of the interface relief function, Eq. 1. Then Eq. 60 leads directly to the equation for the roughness scattering cross section, Eq. 25. Thus, the purpose of this appendix is to outline the derivation of the roughness scattering amplitude and, particularly, Eq. 27. As discussions of this problem have been given by others [4, 5], this appendix will only give a brief outline that emphasizes the physics and establishes notation convenient for computation and consistent with the volume scattering case.

The scattering amplitude is determined by imposing the boundary conditions, Eqs. 6 and 8, on the fields. The derivative operations that yield the displacement and shear are readily implemented in the plane-wave expansions, where derivatives become multiplication. For example, the displacement in the water is

$$\mathbf{u}^{(f)}(\mathbf{r}) = i \int \frac{d^2K}{\nu_f(\mathbf{K})}\ \mathbf{e}_f^{+}(\mathbf{K}) A_f(\mathbf{K},\mathbf{K}_i) \exp(i\mathbf{k}_f^{+}(\mathbf{K})\cdot\mathbf{r}) \quad , \tag{62}$$

and the displacement due to shear in the sea bed is

$$\mathbf{u}^{(t)}(\mathbf{r}) = i a_t^{-1} \int \frac{d^2K}{\nu_t(\mathbf{K})}\ [\mathbf{e}_v^{-}(\mathbf{K}) A_v(\mathbf{K},\mathbf{K}_i) + \mathbf{e}_h^{-}(\mathbf{K}) A_h(\mathbf{K},\mathbf{K}_i)] \exp(i\mathbf{k}_t^{-}(\mathbf{K})\cdot\mathbf{r}) \tag{63}$$

The additional derivatives needed to form the stress tensor introduce additional wave vector factors, which are taken as unit vectors multiplied by wave numbers. The derivative factors comprise the matrix given in Eq. 32. The first column of $E^{(n)}(\mathbf{K})$ pertains to the in-water scattering amplitude, while the second, third, and fourth pertain to the p, v, and h fields in the sea bed material. The fifth column pertains to the incident field. The boundary conditions require that the differences in normal displacement and normal tractions vanish across the boundary, accordingly, the matrix $E^{(n)}(\mathbf{K})$ contains appropriate sign reversals. The first three rows ($m = 1, 2, 3$) of $E^{(n)}(\mathbf{K})$ express the derivatives needed for the stress tensor, σ_{mn}, and the last row expresses the derivatives needed for the nth component of displacement. That is, considering the plane-wave expansion

$$X^{(n)} = \int d^2K\ \ E^{(n)}(\mathbf{K}) Y_1^{-1}(\mathbf{K}) U \quad , \tag{64}$$

with

$$U = \begin{pmatrix} A_f(\mathbf{K},\mathbf{K}_i) \exp(i\mathbf{k}_f^{+}(\mathbf{K})\cdot\mathbf{r}) \\ A_p(\mathbf{K},\mathbf{K}_i) \exp(i\mathbf{k}_t^{-}(\mathbf{K})\cdot\mathbf{r}) \\ A_v(\mathbf{K},\mathbf{K}_i) \exp(i\mathbf{k}_t^{-}(\mathbf{K})\cdot\mathbf{r}) \\ A_h(\mathbf{K},\mathbf{K}_i) \exp(i\mathbf{k}_t^{-}(\mathbf{K})\cdot\mathbf{r}) \\ k_f \nu_f(\mathbf{K}) \delta(\mathbf{K}-\mathbf{K}_i) \exp(i\mathbf{k}_0^{-}(\mathbf{K})\cdot\mathbf{r}) \end{pmatrix} , \tag{65}$$

the mth row of $X^{(n)}$ ($m = 1, 2, 3$) is proportional to the water-sea bed difference in the mn-component of stress, and the 4th row is proportional to the water-sea bed difference in the nth component of the displacement vector.

The boundary conditions can now be written as

$$X^{(1)}\partial_1\zeta(\mathbf{R}) + X^{(2)}\partial_2\zeta(\mathbf{R}) - X^{(3)} = 0 \quad , \tag{66}$$

with $x_3 = \zeta(\mathbf{K})$. To obtain the zeroth- and first-order solutions, the scattering amplitudes are expressed as a sum of zeroth- and first-order terms, the former being the flat-surface solution and the latter being linear functionals of $\zeta(\mathbf{R})$.

$$A_\alpha(\mathbf{K}, \mathbf{K}_i) = A_\alpha^{(0)}(\mathbf{K}, \mathbf{K}_i) + A'_\alpha(\mathbf{K}, \mathbf{K}_i) \quad . \tag{67}$$

Also, the exponentials used are expanded to first order in $\zeta(\mathbf{R})$ with $x_3 = \zeta(\mathbf{R})$,

$$\exp(i\mathbf{k}_\alpha^\pm(\mathbf{K}) \cdot \mathbf{r}) = \exp(i\mathbf{K} \cdot \mathbf{R})[1 \pm ik_f\nu_\alpha(\mathbf{K})\zeta(\mathbf{R}) + \cdots] \quad . \tag{68}$$

Finally, a Fourier transform of Eq. 66 with respect to $\mathbf{R}$ is taken and the definition

$$Z(\mathbf{K}) = \frac{1}{(2\pi)^2}\int d^2R \exp(-i\mathbf{K} \cdot \mathbf{R})\zeta(\mathbf{R}) \tag{69}$$

is used, and the derivatives of $\zeta(\mathbf{R})$ in Eq. 66 are treated using

$$i\mathbf{K}Z(\mathbf{K}) = \frac{1}{(2\pi)^2}\int d^2R \exp(-i\mathbf{K} \cdot \mathbf{R})\nabla\zeta(\mathbf{R}) \quad . \tag{70}$$

The zeroth-order terms yield

$$P^{(3)}(\mathbf{K}_i)A^{(0)}(\mathbf{K}_s, \mathbf{K}_i) + Q^{(3)}(\mathbf{K}_i)\delta(\mathbf{K}_s - \mathbf{K}_i) = 0 \quad , \tag{71}$$

where $A^{(0)}(\mathbf{K}_s, \mathbf{K}_i)$ is a column matrix comprised of the zeroth-order scattering amplitudes. Equation 71 is equivalent to Eq. 41.

The equation for first-order terms is

$$ik_f^2B(\mathbf{K}_s, \mathbf{K}_i)D_0(\mathbf{K}_i)Z(\mathbf{K}_s - \mathbf{K}_i) - P^{(3)}(\mathbf{K}_s)Y_1^{-1}A'(\mathbf{K}_s, \mathbf{K}_i) = 0, \tag{72}$$

where $B(\mathbf{K}_s, \mathbf{K}_i)$ is given by Eq. 29 and $A'(\mathbf{K}_s, \mathbf{K}_i)$ is a column matrix comprised of the first-order approximation to the incoherent scattering amplitudes. Equation 72 is equivalent to Eqs. 27 and 61.

Appendix C: Volume Scattering

This appendix outlines first-order perturbation theory (Born approximation) for scattering from an inhomogeneous elastic half space with planar boundary and fluid above. This treatment is the same in principle, but less detailed than that given by Ivakin [13]. This appendix uses different notation than Ref. [13], and a more general expression for the bistatic cross section is developed here, in a form convenient for computation.

Consider small fluctuations of elastic medium parameters and corresponding corrections to the displacement field and potentials, making corresponding change in the wave equations

$$\rho, \lambda, \mu, \mathbf{u}, \phi, \vec{\psi} \rightarrow \rho + \rho', \lambda + \lambda', \mu + \mu', \mathbf{u} + \mathbf{u}', \phi + \phi', \vec{\psi} + \vec{\psi}'.$$

This gives, for the zero-order potentials, the following well-known wave equations for a homogeneous elastic medium

$$(\nabla^2 + k_p^2)\phi = 0,$$

$$(\nabla^2 + k_t^2)\vec{\psi} = 0,$$

and the following equation for the first-order field perturbation in the displacement

$$\rho\omega^2\mathbf{u}' + \rho c_p^2 \nabla(\nabla \cdot \mathbf{u}') - \rho c_t^2 \nabla \times \nabla \times \mathbf{u}' = \mathbf{f},$$

where $\mathbf{f}$ is a vector with components

$$f_i = -\rho'\omega^2 u_i - \partial_i \left(\lambda'(\nabla \cdot \mathbf{u})\right) - \sum_j \partial_j \left(\mu'(\partial_i u_j + \partial_j u_i)\right), \; i, j = 1, 2, 3. \quad (73)$$

This corresponds to the following pair of first-order equations for the perturbed potentials:

$$\rho(\omega^2 + c_p^2\nabla^2)\nabla^2\phi' = \nabla \cdot \mathbf{f}$$

$$\rho(\omega^2 + c_t^2\nabla^2)\nabla \times \nabla \times \vec{\psi}' = \nabla \times \mathbf{f} \,.$$

A solution of these equations is

$$\phi'(\mathbf{r}) = -\frac{1}{\rho\omega^2}\int G_p(\mathbf{r}-\mathbf{r}')\nabla\cdot\mathbf{f}(\mathbf{r}')d^3r',$$

$$\vec{\psi}'(\mathbf{r}) = \frac{1}{\rho\omega^2}\int G_t(\mathbf{r}-\mathbf{r}')\nabla\times\mathbf{f}(\mathbf{r}')d^3r',$$

where $G_\alpha(\mathbf{r})\,(\alpha = p,t\ )$ are free-space Green's functions, which can be expressed as expansions in plane waves propagating up toward the interface

$$G_\alpha(\mathbf{r}) = -\frac{1}{4\pi r}\exp(ik_\alpha r) = -\frac{i}{8\pi^2}\int\frac{d^2K}{k_f\nu_\alpha(\mathbf{K})}\ \exp\left(i\mathbf{k}_\alpha^+(\mathbf{K})\cdot\mathbf{r}\right).$$

Then the scattered fields which are incident on the interface can be also expressed in terms of plane wave expansions

$$\phi'(\mathbf{r}) = \int\frac{d^2K}{k_f\nu_p(\mathbf{K})}\ A'_p(\mathbf{K},\mathbf{K}_i)\ \exp\left(i\mathbf{k}_p^+(\mathbf{K})\cdot\mathbf{r}\right),$$

$$\vec{\psi}'(\mathbf{r}) = \int\frac{d^2K}{k_f\nu_t(\mathbf{K})}\ \mathbf{A}'_t(\mathbf{K},\mathbf{K}_i)\ \exp\left(i\mathbf{k}_t^+(\mathbf{K})\cdot\mathbf{r}\right),$$

where

$$A'_p = \frac{i\pi}{\rho\omega^2}F_{\mathbf{k}_p}(\nabla\cdot\mathbf{f}),$$

$$\mathbf{A}'_t = -\frac{i\pi}{\rho\omega^2}F_{\mathbf{k}_t}(\nabla\times\mathbf{f}),$$

$$F_\mathbf{k}(Q) = (2\pi)^{-3}\int Q(\mathbf{r})\exp(-i\mathbf{k}\cdot\mathbf{r})d^3r. \tag{74}$$

Transmission of these waves through the flat interface is governed by the expression

$$\frac{A'_f(\mathbf{K},\mathbf{K}_i)}{\nu_0(\mathbf{K})} = W_p^+(\mathbf{K})\frac{A'_p(\mathbf{K},\mathbf{K}_i)}{\nu_p(\mathbf{K})} + W_t^+(\mathbf{K})\frac{\mathbf{e}_h(\mathbf{K})\cdot\mathbf{A}'_t(\mathbf{K},\mathbf{K}_i)}{\nu_t(\mathbf{K})},$$

where the W_α^+ are the transmission coefficients for the upward direction and are related to downward direction coefficients, W_α, as follows

$$W_\alpha^+/\nu_\alpha = a_\rho W_\alpha/\nu_f,\ \alpha = p, t.$$

Note that $W_t = W_v$.

Then, using the zero-order solution in Eq. 73, one obtains the following expression for the scattering amplitude

$$A'_f(\mathbf{K}, \mathbf{K}_i) = -i\pi k_f^2 m \sum_{\eta,\beta} d_{\eta\beta} F_{\mathbf{q}_\eta}(\epsilon_\beta). \tag{75}$$

with $\mathbf{q}_\eta$ as defined by Eqs.44-46 and $d_{\eta\beta}$ as defined by Eqs. 47-49. Using Eqs. 74,75, the expression for the cross section, Eq. 43, can be readily obtained.

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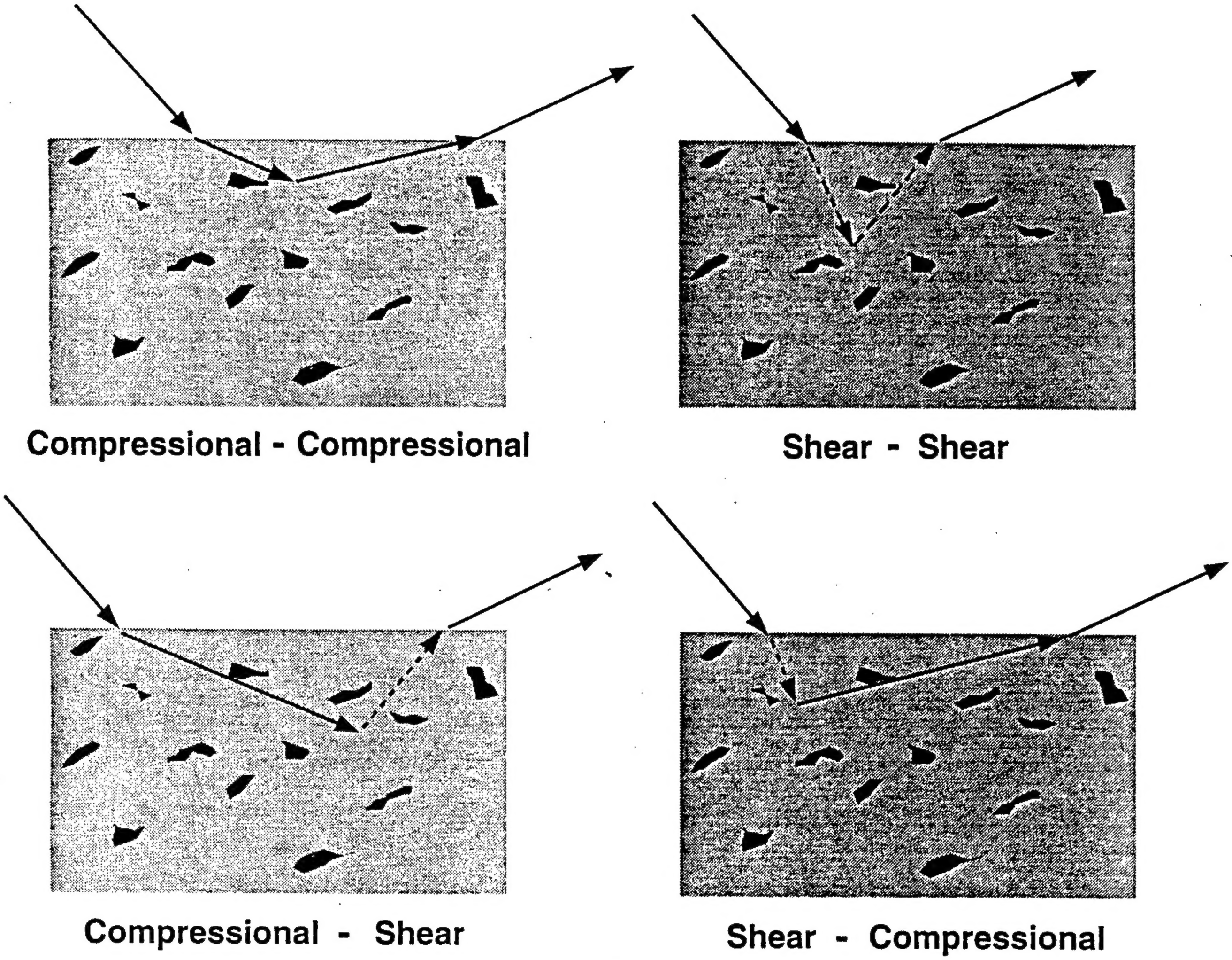
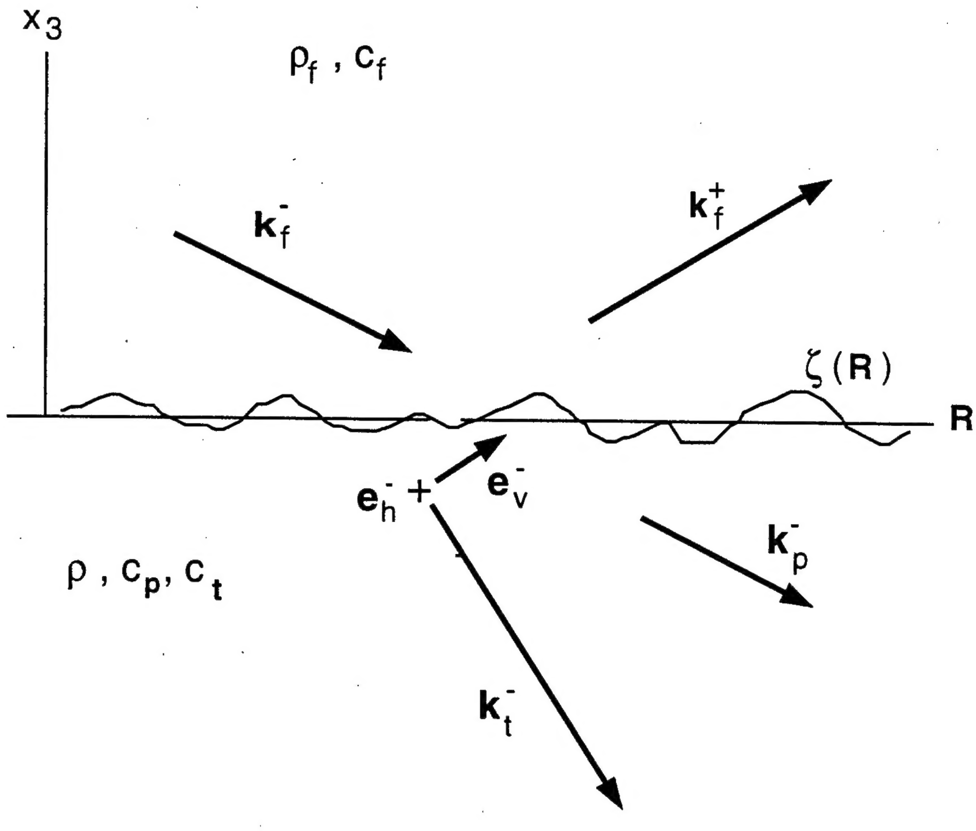

Compressional - Compressional
Shear - Shear
Compressional - Shear
Shear - Compressional

Fig. 1



x_3
ρ_f , c_f
$\mathbf{k}_f^-$
$\mathbf{k}_f^+$
$\zeta(\mathbf{R})$
$\mathbf{R}$
$\mathbf{e}_h^- + \mathbf{e}_v^-$
$\mathbf{k}_p^-$
ρ , $c_\mathbf{p}$, $c_\mathbf{t}$
$\mathbf{k}_t^-$

Fig. 2

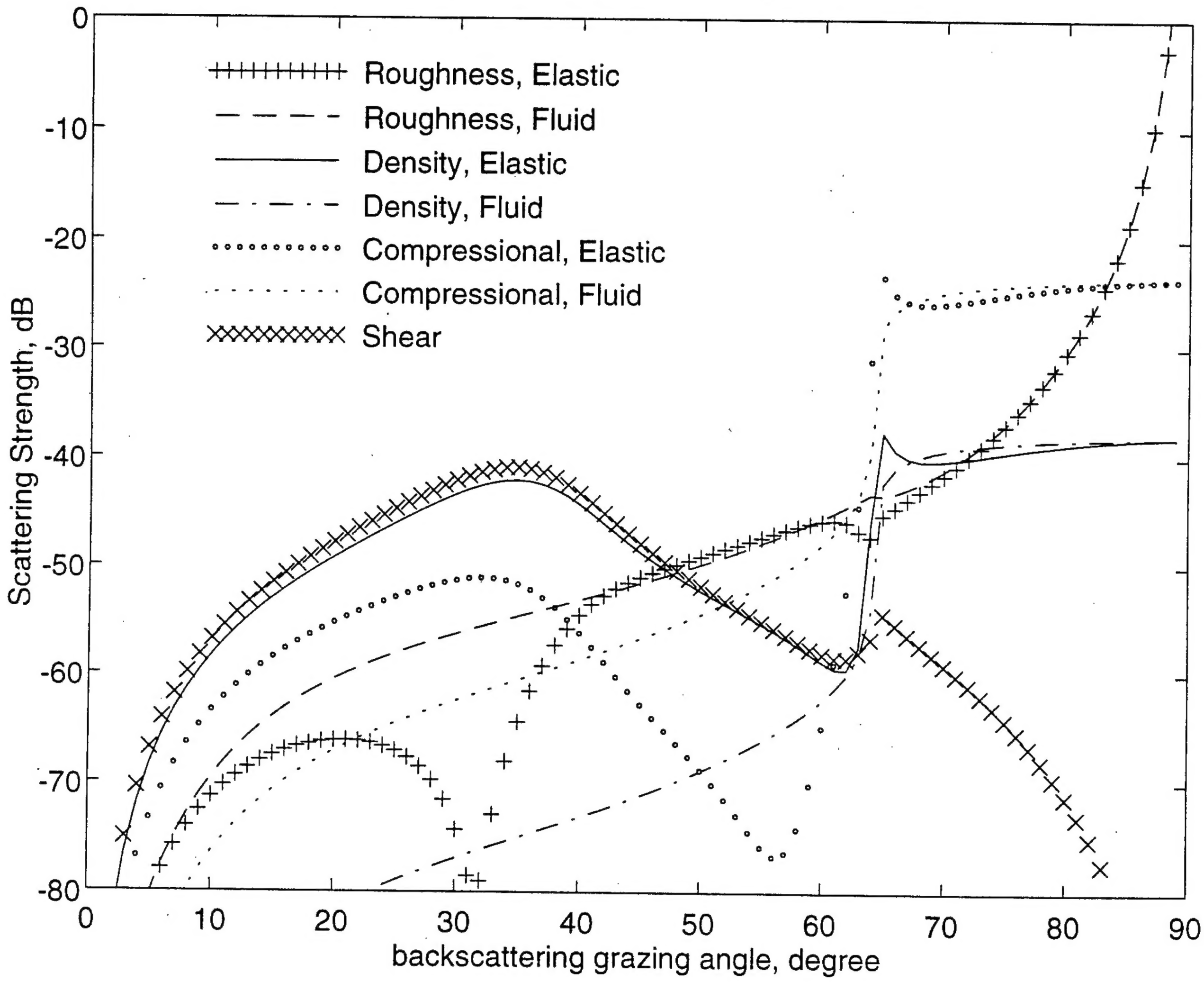

Roughness, Elastic
Roughness, Fluid
Density, Elastic
Density, Fluid
Compressional, Elastic
Compressional, Fluid
Shear
Scattering Strength, dB
backscattering grazing angle, degree
0
-10
-20
-30
-40
-50
-60
-70
-80
0
10
20
30
40
50
60
70
80
90

Fig. 3